TA Session 1

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Outline

GMM estimation review

Moment Conditions

Let $g_i()$ be a known $l \times 1$ function of the i^{th} observation and a $k \times 1$ parameter β. A moment equation model is summarized by the moment equations.

$$E[g_i(\beta)] = 0$$

Identification condition: $l \geq k$

Method of Moments Estimator

By the WLN we know that

$$1/n \sum_{i=1}^{n} g_i(\beta) \underset{p}{\rightarrow} E[g_i(\beta)]$$

Therefore, the method of moment estimator is

$$1/n \sum_{i=1}^{n} g_i(\hat{\beta}_{gmm}) = \bar{g}_n(\hat{\beta}_{gmm}) = 0$$

GMM ESTIMATOR

Let

$$J(\beta) = \bar{g}_n(\beta)' W \bar{g}_n(\beta)'$$

The GMM estimator is given by

$$\hat{\beta}_{gmm} = \arg\min J(\beta)$$

Asymptotic properties

- Consistency: $\hat{\beta}_{gmm} \underset{p}{\rightarrow} \beta$
- Asymptotic Distribution:

$$\sqrt{n}(\hat{\beta}_{gmm} - \beta) \underset{d}{\rightarrow} N(0, V_\beta)$$
$$V_\beta = (Q'WQ)^{-1}(Q'W\Omega WQ)(Q'WQ)^{-1}$$

Efficient GMM

Let $W = \Omega^{-1}$ then

$$\begin{aligned} V_\beta &= (Q'WQ)^{-1}(Q'W\Omega WQ)(Q'WQ)^{-1} \\ &= (Q'\Omega^{-1}Q)^{-1} \end{aligned}$$

Which is the minimum variance of β

TWO-STEP GMM

Note that we can define a estimator for the variance such that $\hat{\Omega} \underset{p}{\rightarrow} \Omega$.

Consequently, to obtain the Efficient GMM estimator we perform the following steps:

1) Estimate the model using some weighting matrix (for example $W = I$) and compute $\hat{\Omega}$

2) Using $W = \hat{\Omega}^{-1}$, re-estimate the model

Empirical Work

INTRODUCTION

- I will present the paper Household Sharing and Commitment: Evidence from Panel Data on Individual Expenditures and Time Use from Jeremy Lise and Ken Yamada published on The Review of Economic Studies (2019)
- In this article, the authors present novel results on how the allocations of time and individual consumption expenditures differ between households, as well as how these allocations change over time within households
- These patterns are informative for understanding how households determine the relative weights attached to the utilities of the wife and the husband
- The data set is a panel of households, observed for up to 20 years, where we observe the private consumption expenditures and time allocation.

Parameters and decision variables

- Each spouse $j \in [H, W]$ in period t cares about his or her own private consumption c_{jt}, private leisure $l_{j,t}$, and a household public goof q_t
- Individuals spend time on market production m_{jt} and home production h_{jt}
- The public good is produced using a combination of market goods g_t and home production.
- Wives and husbands have different utilities functions $u_t^j(.)$
- Preferences are additively separable over time with discount factor δ_j
- The weight of the wife's utility is given by μ_t

Maximization Problem

$$\max U_0 = E\left[\sum_{t=0}^{T} \delta_W^t \mu_t u_t^W(c_{Wt}, l_{Wt}, q_t) + \delta_H^t (1-\mu_t) u_t^H(c_{Ht}, l_{Ht}, q_t)\right]$$

$$\text{s.t. } q_t = q(g_t, h_{Wt}, h_{Ht}) \quad (1)$$

$$l_{jt} + m_{ht} + h_{jt} = 1 \text{ for } j \in W, H \quad (2)$$

$$c_{Wt} + c_{Ht} + g_t + W_{Wt}(l_{Wt} + h_{Wt}) + W_{Ht}(l_{Ht} + h_{Ht})$$

$$= W_{Wt} + W_{Ht} + (1 + r_t)a_t - a_{t+1} \equiv y_t \quad (3)$$

$$c_{jt}, g_{jt}, l_{jt}, h_{jt}, m_{jt} \geq 0 \quad (4)$$

$$log(W_{jt}) = W(x_{jt}, \epsilon_{jt}) \quad (5)$$

JPSC DATA

The JPSC is a panel dataset from Japan covering period from 1993 to 2013 where we observe:

- private consumption of the wife, the husband and the household
- hours each member spend on market work, home production and leisure
- individual wages
- household characteristics
- demographic information (age, age at marriage and education)

Summary Statistics

TABLE 1
Summary statistics JPSC 1993–2013

	Mean [standard deviation]		
	Wife	Husband	Household
Expenditure per month	36,711 [63,951]	77,650 [64,321]	442,640 [275,174]
(% of household total)	(6.5%)	(14.5%)	(79.0%)
Time use, hours per week			
(share of own time)			
Market work	29.7h [21.7]	62.7h [14.7]	
- including commuting	(17.7%)	(37.3%)	
Home production	44.0h [25.3]	7.4h [9.1]	
- including child care	(26.2%)	(4.4%)	
Leisure	94.2h [20.1]	97.9h [15.6]	
- including sleep	(56.1%)	(58.3%)	
Observables			
Age	36.5 [6.4]	38.9 [7.4]	
Education (years)	13.2 [1.6]	13.5 [2.3]	
Wage	925 [1,014]	1,676 [1,575]	
Children aged 0–6			0.62 [0.80]
Household size			4.24 [1.49]

Notes: All monetary values are in 2013 Japanese Yen. The sample comprises 1,149 households.

Parametrization

- utility function

$$u^j(c_{jt}, l_{jt}, q_t) = \frac{\xi_t^j}{1-\sigma^j}\left(\alpha_{1t}^j c_{jt}^{\phi^j} + \alpha_{2t}^j l_{jt}^{\phi^j} + (1-\alpha_{1t}^j - \alpha_{2t}^j) q_t^{\phi^j}\right)^{\frac{1-\sigma^j}{\phi^j}}$$

- home production function

$$q(h_{Wt}, h_{Ht}, g_t) = (\pi_t h_{Wt}^{\gamma} + (1-\pi_t) h_{Ht}^{\gamma})^{\frac{\rho}{\gamma}} g_t^{1-\rho}$$

INTRATEMPORAL CONDITIONS

- The authors uses only the intratemporal conditions since these conditions are robust to liquidity constraints
- **Home production technology**

$$\left(\frac{\pi_t}{1-\pi_t}\right)\left(\frac{h_{Wt}}{h_{Ht}}\right)^{\gamma-1} = \frac{W_{Wt}}{W_{Ht}} \tag{6}$$

$$\pi_t\left(\frac{\rho}{1-\rho}\right)\left(\frac{h_{Wt}^{\gamma-1}}{G_t}\right)g_t = W_{Wt} \tag{7}$$

$$(1-\pi_t)\left(\frac{\rho}{1-\rho}\right)\left(\frac{h_{Ht}^{\gamma-1}}{G_t}\right)g_t = W_{Ht} \tag{8}$$

where $G_t = \pi_t h_{At}^{\gamma} + (1-\pi_t)h_{Bt}^{\gamma}$

Intratemporal conditions

- **Private Consumption and Leisure**

$$\frac{\alpha^j_{1t}}{\alpha^j_{2t}}\left(\frac{c_{jt}}{l_{jt}}\right)^{\phi^j-1} = \frac{1}{W_{jt}} \tag{9}$$

$$\left(\frac{\mu_t}{1-\mu_t}\right)\left(\frac{A_{Wt}^{\frac{1-\sigma^W-\phi^W}{\phi^W}}\alpha^W_{1t}c_{Wt}^{\phi^W-1}}{A_{Ht}^{\frac{1-\sigma^H-\phi^H}{\phi^H}}\alpha^H_{1t}c_{Ht}^{\phi^H-1}}\right)\left(\frac{\xi^W_t}{\xi^H_t}\right) = 1 \tag{10}$$

$$\left(\frac{\mu_t}{1-\mu_t}\right)\left(\frac{A_{Wt}^{\frac{1-\sigma^W-\phi^W}{\phi^W}}\alpha^W_{2t}l_{Wt}^{\phi^W-1}}{A_{Ht}^{\frac{1-\sigma^H-\phi^H}{\phi^H}}\alpha^H_{2t}l_{Ht}^{\phi^H-1}}\right)\left(\frac{\xi^W_t}{\xi^H_t}\right) = \frac{W_{Wt}}{W_{Ht}} \tag{11}$$

where $A_{jt} = \alpha^j_{1t}c_{jt}^{\phi^j} + \alpha^j_{2t}l_{jt}^{\phi^j} + (1-\alpha^j_{1t}-\alpha^j_{2t})q_t^{\phi^j}$

Intratemporal conditions

- **Public Consumption**

$$\mu_t^j \xi_t^j A_{jt}^{\frac{1-\sigma^j-\phi^j}{\phi^j}} \alpha_{2t}^j l_{jt}^{\phi^j-1} = \pi_t \rho h_{jt}^{\gamma^j-1} G_t^{\frac{\rho-\gamma}{\gamma}} g_t^{1-\rho} D_t \tag{12}$$

$$\mu_t^j \xi_t^j A_{jt}^{\frac{1-\sigma^j-\phi^j}{\phi^j}} \alpha_{1t}^j c_{jt}^{\phi^j-1} = (1-\rho) G_t^{\frac{\rho}{\gamma}} g_t^{-\rho} D_t \tag{13}$$

$$\text{where } D_t = \sum_{j \in [W,H]} \mu_t^j \xi_t^j A^{\frac{1-\sigma^j-\phi^j}{\phi^j}} (1-\alpha_{1t}^j - \alpha_{2t}^j) q_t^{\phi^j-1} \tag{14}$$

Heterogeneity

- The authors parameterize the heterogeneity in preferences and home production in terms of observable variables (x_{Wt}, x_{Ht}).
- The Pareto weight is parametrized in terms of observable distribution factors (z_0, z_{1t})
- **Preference Heterogeneity**

$$\alpha_{kt}^{j} = \frac{\exp\left(\alpha_k^{j\prime} x_{jt}\right)}{1 + \exp\left(\alpha_1^{j\prime} x_{jt}\right) + \exp\left(\alpha_2^{j\prime} x_{jt}\right)} \text{ for } k = 1, 2 \tag{15}$$

where x_{jt} contains a constant, age and the number of children in the household

HETEROGENEITY

- **Home Production Heterogeneity**

$$\pi_t = \frac{\exp(\pi' x_t)}{1 + \exp(\pi' x_t)} \tag{16}$$

$$\rho = \frac{\exp(\rho_0)}{1 + \exp(\rho_0)} \tag{17}$$

where x_t contains a constant and the number of children in the household under the age of seven

Wage Process

We assume that wages evolve as a first-order autoregressive process with individual fixed effects

$$\log W_{jt} = \vartheta^j + \theta_1^j a_{jt} + \theta_2^j a_{jt}^2 + \epsilon_{jt} \tag{18}$$

$$\epsilon_{jt} = \varrho_{jt} + e_{jt}$$

$$\varrho_{jt} = \varrho_{j,t-1} + \nu_{jt}, \varrho_{j,-1} = 0$$

where ϑ^j is an individual fixed effect and a_{jt} is potential experience
The unobservable is ϵ_{jt} and comprises a permanent component ϱ_{jt} and a measurement error e_{jt}

PARETO WEIGHT

$$\mu_t = \frac{\exp(\mu_0' z_0 + \mu_1' z_{1t})}{1 + \exp(\mu_0' z_0 + \mu_1' z_{1t})} \tag{19}$$

Where z_0 are distribution factors known or forecast at the time of marriage and, $z_{1t} \equiv z_t + E_0 z_t$ is the realizes deviation from this time zero prediction.

GMM

- Using the equations from previous slides we can create the the estimation equations.
- Consumption and hours are endogenous variables correlated with unobserved preference shocks
- Wages are assumed to be measure with error
- To instrument for the endogenous variables and the measurement error we use the levels of consumption, hours and wages in the intratemporal conditions.
- Observable preferences and productivity shifters, as well as observable distribution factors are treated as exogenous
- To estimate the wage process is used the Heckman (1979) two step procedure

ESTIMATES

TABLE 2
Estimates

	I	II	III	IV	V	VI	VII	VIII	IX
Home production									
γ	0.682	0.676	0.647	0.650	0.682	0.667	0.682	0.678	0.606
	(0.037)	(0.036)	(0.042)	(0.042)	(0.037)	(0.038)	(0.037)	(0.037)	(0.044)
π (at sample mean)	0.459	0.462	0.471	0.471	0.456	0.464	0.459	0.461	0.487
	(0.018)	(0.017)	(0.020)	(0.020)	(0.018)	(0.018)	(0.018)	(0.018)	(0.020)
ρ	0.080	0.079	0.077	0.078	0.080	0.079	0.079	0.079	0.075
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Preferences									
ϕ^W	0.158	0.162	0.179	0.151	0.161	0.151	0.159	0.164	0.172
	(0.052)	(0.019)	(0.022)	(0.017)	(0.055)	(0.041)	(0.052)	(0.052)	(0.046)
ϕ^H	0.624	0.607	0.576	0.575	0.630	0.607	0.624	0.620	0.564
	(0.039)	(0.040)	(0.067)	(0.065)	(0.039)	(0.040)	(0.039)	(0.040)	(0.049)
α_1^W (at sample mean)	0.184	0.184	0.184	0.179	0.186	0.182	0.184	0.185	0.188
	(0.015)	(0.005)	(0.007)	(0.006)	(0.016)	(0.012)	(0.015)	(0.015)	(0.014)
α_2^W (at sample mean)	0.259	0.255	0.241	0.257	0.258	0.259	0.258	0.255	0.253
	(0.024)	(0.012)	(0.015)	(0.014)	(0.025)	(0.020)	(0.024)	(0.024)	(0.022)
α_1^H (at sample mean)	0.423	0.416	0.405	0.400	0.424	0.415	0.422	0.422	0.396
	(0.021)	(0.017)	(0.026)	(0.025)	(0.021)	(0.019)	(0.020)	(0.020)	(0.023)
α_1^W (at sample mean)	0.180	0.186	0.194	0.193	0.179	0.184	0.181	0.182	0.200
	(0.012)	(0.013)	(0.025)	(0.025)	(0.011)	(0.012)	(0.012)	(0.012)	(0.015)

Estimates

Pareto weight									
μ (at sample mean)	0.438	0.438	0.435	0.437	0.434	0.439	0.438	0.437	0.423
	(0.008)	(0.008)	(0.011)	(0.011)	(0.009)	(0.008)	(0.008)	(0.008)	(0.008)
$\omega_{W,0} - \omega_{H,0}$	0.404	0.407	0.419	0.414	0.401	0.401	0.404	0.405	0.495
	(0.027)	(0.023)	(0.044)	(0.043)	(0.027)	(0.029)	(0.027)	(0.027)	(0.020)
$\Delta\omega_{W,10} - \Delta\omega_{H,10}$	0.306	0.285	0.328	0.286	0.293	0.287	0.306	0.307	0.403
	(0.174)	(0.148)	(0.256)	(0.242)	(0.170)	(0.181)	(0.174)	(0.174)	(0.147)
ν_0	0.028	0.027	0.028	0.028	0.028	0.027	0.027	0.028	0.024
	(0.013)	(0.012)	(0.020)	(0.020)	(0.012)	(0.013)	(0.013)	(0.013)	(0.012)
z_{1t}	0.338					0.328	0.342	0.374	0.439
	(0.015)					(0.016)	(0.016)	(0.021)	(0.031)
Δz_{1t}		0.350	0.362	0.372					
		(0.012)	(0.025)	(0.033)					
$z_{1,t-1}$		0.347	0.359	0.351					
		(0.016)	(0.030)	(0.039)					
$z_{1,t-2}$			0.001						
			(0.022)						
$\Delta z_{1,t+1}$				0.023					
				(0.022)					
$z_{1t} \times \mathbf{1}\{z_{1t} < q_1\}$					0.368				
					(0.026)				
$z_{1t} \times \mathbf{1}\{q_1 \leq z_{1t} < q_2\}$					0.503				
					(0.108)				
$z_{1t} \times \mathbf{1}\{q_2 \leq z_{1t} < q_3\}$					0.057				
					(0.309)				
$z_{1t} \times \mathbf{1}\{q_3 \leq z_{1t} < q_4\}$					0.187				
					(0.103)				
$z_{1t} \times \mathbf{1}\{q_4 \leq z_{1t}\}$					0.290				
					(0.032)				
$z_{1t} \times$ divorce						0.340			
						(0.137)			
$z_{1t} \times$ new child							−0.051		
							(0.055)		
$z_{1t} \times$ wallet sharing								−0.070	
								(0.030)	
$z_{1t} \times \mathbf{1}\{\text{ind}_W \neq \text{ind}_H\}$									−0.063
									(0.037)

Thank you!